

the angle between these two vectors. The calculations of these quantities are:

$$\begin{aligned}\tau &= \vec{\mu} \times \vec{B} = \mu B \sin \theta = (2.5 \times 10^{-6} \text{ A} \cdot \text{m}^2)(0.50 \text{ T}) \sin(30^\circ) = 6.3 \times 10^{-7} \text{ N} \cdot \text{m} \\ U &= -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta = -(2.5 \times 10^{-6} \text{ A} \cdot \text{m}^2)(0.50 \text{ T}) \cos(30^\circ) = -1.1 \times 10^{-6} \text{ J}.\end{aligned}$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.



11.4 Check Your Understanding

In what orientation would a magnetic dipole have to be to produce (a) a maximum torque in a magnetic field? (b) A maximum energy of the dipole?

11.6 | The Hall Effect

Learning Objectives

By the end of this section, you will be able to:

- Explain a scenario where the magnetic and electric fields are crossed and their forces balance each other as a charged particle moves through a velocity selector
- Compare how charge carriers move in a conductive material and explain how this relates to the Hall effect

In 1879, E.H. Hall devised an experiment that can be used to identify the sign of the predominant charge carriers in a conducting material. From a historical perspective, this experiment was the first to demonstrate that the charge carriers in most metals are negative.



Visit this [website \(https://openstaxcollege.org//21halleffect\)](https://openstaxcollege.org//21halleffect) to find more information about the Hall effect.

We investigate the **Hall effect** by studying the motion of the free electrons along a metallic strip of width l in a constant magnetic field (**Figure 11.17**). The electrons are moving from left to right, so the magnetic force they experience pushes them to the bottom edge of the strip. This leaves an excess of positive charge at the top edge of the strip, resulting in an electric field E directed from top to bottom. The charge concentration at both edges builds up until the electric force on the electrons in one direction is balanced by the magnetic force on them in the opposite direction. Equilibrium is reached when:

$$eE = ev_d B \quad (11.24)$$

where e is the magnitude of the electron charge, v_d is the drift speed of the electrons, and E is the magnitude of the electric field created by the separated charge. Solving this for the drift speed results in

$$v_d = \frac{E}{B}. \quad (11.25)$$

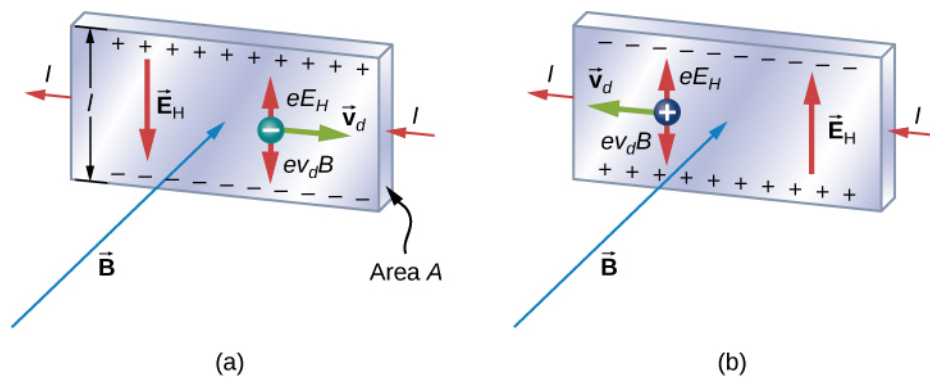


Figure 11.17 In the Hall effect, a potential difference between the top and bottom edges of the metal strip is produced when moving charge carriers are deflected by the magnetic field. (a) Hall effect for negative charge carriers; (b) Hall effect for positive charge carriers.

A scenario where the electric and magnetic fields are perpendicular to one another is called a crossed-field situation. If these fields produce equal and opposite forces on a charged particle with the velocity that equates the forces, these particles are able to pass through an apparatus, called a **velocity selector**, undeflected. This velocity is represented in **Equation 11.26**. Any other velocity of a charged particle sent into the same fields would be deflected by the magnetic force or electric force.

Going back to the Hall effect, if the current in the strip is I , then from **Current and Resistance**, we know that

$$I = nev_d A \quad (11.26)$$

where n is the number of charge carriers per volume and A is the cross-sectional area of the strip. Combining the equations for v_d and I results in

$$I = ne \left(\frac{E}{B} \right) A. \quad (11.27)$$

The field E is related to the potential difference V between the edges of the strip by

$$E = \frac{V}{l}. \quad (11.28)$$

The quantity V is called the Hall potential and can be measured with a voltmeter. Finally, combining the equations for I and E gives us

$$V = \frac{IBl}{neA} \quad (11.29)$$

where the upper edge of the strip in **Figure 11.17** is positive with respect to the lower edge.

We can also combine **Equation 11.24** and **Equation 11.28** to get an expression for the Hall voltage in terms of the magnetic field:

$$V = Blv_d. \quad (11.30)$$

What if the charge carriers are positive, as in **Figure 11.17**? For the same current I , the magnitude of V is still given by **Equation 11.29**. However, the upper edge is now negative with respect to the lower edge. Therefore, by simply measuring the sign of V , we can determine the sign of the majority charge carriers in a metal.

Hall potential measurements show that electrons are the dominant charge carriers in most metals. However, Hall potentials indicate that for a few metals, such as tungsten, beryllium, and many semiconductors, the majority of charge carriers are positive. It turns out that conduction by positive charge is caused by the migration of missing electron sites (called holes) on ions. Conduction by holes is studied later in **Condensed Matter Physics** (<http://cnx.org/content/m58591/latest/>).

The Hall effect can be used to measure magnetic fields. If a material with a known density of charge carriers n is placed in a magnetic field and V is measured, then the field can be determined from **Equation 11.29**. In research laboratories where the fields of electromagnets used for precise measurements have to be extremely steady, a “Hall probe” is commonly used as part of an electronic circuit that regulates the field.

Example 11.8

Velocity Selector

An electron beam enters a crossed-field velocity selector with magnetic and electric fields of 2.0 mT and 6.0×10^3 N/C, respectively. (a) What must the velocity of the electron beam be to traverse the crossed fields undeflected? If the electric field is turned off, (b) what is the acceleration of the electron beam and (c) what is the radius of the circular motion that results?

Strategy

The electron beam is not deflected by either of the magnetic or electric fields if these forces are balanced. Based on these balanced forces, we calculate the velocity of the beam. Without the electric field, only the magnetic force is used in Newton’s second law to find the acceleration. Lastly, the radius of the path is based on the resulting circular motion from the magnetic force.

Solution

- a. The velocity of the unperturbed beam of electrons with crossed fields is calculated by **Equation 11.25**:

$$v_d = \frac{E}{B} = \frac{6 \times 10^3 \text{ N/C}}{2 \times 10^{-3} \text{ T}} = 3 \times 10^6 \text{ m/s.}$$

- b. The acceleration is calculated from the net force from the magnetic field, equal to mass times acceleration. The magnitude of the acceleration is:

$$ma = qvB$$

$$a = \frac{qvB}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(3 \times 10^6 \text{ m/s})(2 \times 10^{-3} \text{ T})}{9.1 \times 10^{-31} \text{ kg}} = 1.1 \times 10^{15} \text{ m/s}^2.$$

- c. The radius of the path comes from a balance of the circular and magnetic forces, or **Equation 11.25**:

$$r = \frac{mv}{qB} = \frac{(9.1 \times 10^{-31} \text{ kg})(3 \times 10^6 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(2 \times 10^{-3} \text{ T})} = 8.5 \times 10^{-3} \text{ m.}$$

Significance

If electrons in the beam had velocities above or below the answer in part (a), those electrons would have a stronger net force exerted by either the magnetic or electric field. Therefore, only those electrons at this specific velocity would make it through.

Example 11.9

The Hall Potential in a Silver Ribbon

Figure 11.18 shows a silver ribbon whose cross section is 1.0 cm by 0.20 cm. The ribbon carries a current of 100 A from left to right, and it lies in a uniform magnetic field of magnitude 1.5 T. Using a density value of $n = 5.9 \times 10^{28}$ electrons per cubic meter for silver, find the Hall potential between the edges of the ribbon.

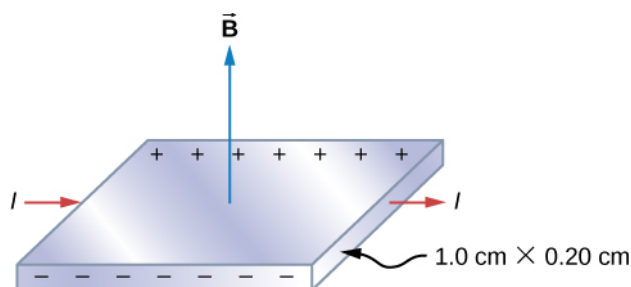


Figure 11.18 Finding the Hall potential in a silver ribbon in a magnetic field is shown.

Strategy

Since the majority of charge carriers are electrons, the polarity of the Hall voltage is that indicated in the figure. The value of the Hall voltage is calculated using **Equation 11.29**:

$$V = \frac{IBl}{neA}.$$

Solution

When calculating the Hall voltage, we need to know the current through the material, the magnetic field, the length, the number of charge carriers, and the area. Since all of these are given, the Hall voltage is calculated as:

$$V = \frac{IBl}{neA} = \frac{(100 \text{ A})(1.5 \text{ T})(1.0 \times 10^{-2} \text{ m})}{(5.9 \times 10^{28} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})(2.0 \times 10^{-5} \text{ m}^2)} = 7.9 \times 10^{-6} \text{ V}.$$

Significance

As in this example, the Hall potential is generally very small, and careful experimentation with sensitive equipment is required for its measurement.



11.5 Check Your Understanding A Hall probe consists of a copper strip, $n = 8.5 \times 10^{28}$ electrons per cubic meter, which is 2.0 cm wide and 0.10 cm thick. What is the magnetic field when $I = 50 \text{ A}$ and the Hall potential is (a) $4.0 \mu\text{V}$ and (b) $6.0 \mu\text{V}$?

11.7 | Applications of Magnetic Forces and Fields

Learning Objectives

By the end of this section, you will be able to:

- Explain how a mass spectrometer works to separate charges
- Explain how a cyclotron works

Being able to manipulate and sort charged particles allows deeper experimentation to understand what matter is made of. We first look at a mass spectrometer to see how we can separate ions by their charge-to-mass ratio. Then we discuss cyclotrons as a method to accelerate charges to very high energies.

Mass Spectrometer

The **mass spectrometer** is a device that separates ions according to their charge-to-mass ratios. One particular version, the Bainbridge mass spectrometer, is illustrated in **Figure 11.19**. Ions produced at a source are first sent through a velocity selector, where the magnetic force is equally balanced with the electric force. These ions all emerge with the same speed $v = E/B$ since any ion with a different velocity is deflected preferentially by either the electric or magnetic force, and